

EFFECT OF BEAMSTRAHLUNG ON BUNCH LENGTH AND EMITTANCE IN FUTURE CIRCULAR e^+e^- COLLIDERS*

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Abstract

In future circular e^+e^- colliders, beamstrahlung may limit the beam lifetime at high energies, and increase the energy spread and bunch length at low energies. If the dispersion or slope of the dispersion is not zero at the collision point, beamstrahlung will also affect the transverse emittance. In this paper, we first examine the beamstrahlung properties, and show that for the proposed FCC-ee, the radiation is fairly well modelled by the classical formulae describing synchrotron radiation in bending magnets. We then derive a set of equations determining the equilibrium emittances in the presence of a nonzero dispersion at the collision point. An example case from FCC-ee will serve as an illustration.

INTRODUCTION

Energy quantization in synchrotron radiation has significant effects in circular colliders, where energy radiated from charged particles is emitted as a discrete random process. The typical time to emit a photon is of order $\rho/(\gamma c)$, where ρ denotes the radius of curvature of a particle trajectory, c the speed of light, and γ the relativistic Lorentz factor. Compared to the betatron and synchrotron periods, we can consider the emission time as instantaneous.

In most electron storage rings, the equilibrium transverse emittances, energy spread and bunch length are determined by a balance of quantum excitation and radiation damping, both occurring in the accelerator bending magnets [1].

A different type of synchrotron radiation, known as beamstrahlung [2, 3, 4, 5, 6], is encountered during the collision with the opposite beam. For short bunch lengths and small transverse beam sizes, the effective bending radius due to the field of the opposing bunch is exceptionally small, and at high energies the classical critical photon energy can become significant. In proposed linear colliders it may even reach the same order of magnitude as the beam energy [6]. In such situations a fully quantum-mechanical description of the synchrotron radiation process could be warranted.

If the energy of an emitted photon is a few percent of its initial energy, the emitting particle may fall outside of the momentum acceptance and be lost. The resulting reduction of the beam lifetime due to the high-energy tail

of the beamstrahlung sets a limit on the quantity $A_\tau \equiv N_b \gamma / (\eta \sigma_x \sigma_x)$ [7, 8], where N_b denotes the bunch charge, σ_x the rms horizontal beam size at the collision point, σ_z the rms bunch length, and η the relative momentum acceptance of the storage ring. A necessary condition for an acceptable beam lifetime is $A_\tau > 10^{28} \text{ m}^{-2}$.

The cumulative effect of statistically independent discontinuous energy changes, introduces a noise excitation of the longitudinal and transverse oscillations, causing their amplitudes to grow until balanced, on average, by the radiation damping. This damping depends only on the average rate of emission of energy and not on any of its other statistical properties, whereas the excitation is due to the fluctuation of the radiation about its average rate.

BEAMSTRAHLUNG

The strength of synchrotron radiation is characterized by the parameter Υ , defined as [5, 6] $\Upsilon \equiv B/B_c = (2/3)\hbar\omega_c/E_e$, with $B_c = m_e^2 c^2 / (e\hbar) \approx 4.4 \text{ GT}$ the Schwinger critical field, ω_c the critical energy as defined by Sands [1], and E_e the electron energy before radiation.

For the collision of 3-dimensional Gaussian bunches with rms sizes σ_x , σ_y and σ_z the peak and average values of Υ are given by [6]

$$\Upsilon_{\max} = 2 \frac{r_e^2 \gamma N_b}{\alpha \sigma_z (\sigma_x^* + \sigma_y^*)}, \quad (1)$$

$$\Upsilon_{\text{ave}} \approx \frac{5}{6} \frac{r_e^2 \gamma N_b}{\alpha \sigma_z (\sigma_x^* + \sigma_y^*)}, \quad (2)$$

where α denotes the fine structure constant ($\alpha \approx 1/137$), and $r_e \approx 2.8 \times 10^{-15} \text{ m}$ the classical electron radius.

Introducing $y \equiv \omega/E_e$ and

$$\xi \equiv \frac{2\omega}{3\Upsilon(E - \hbar\omega)}, \quad (3)$$

the emission rate spectrum (photons emitted per second per energy interval) is described by the functions [6],

$$\frac{dW_\gamma}{d\omega\hbar} = \frac{\alpha}{\sqrt{3}\hbar\pi\gamma^2} \left(\int_\xi^\infty K_{5/3}(\xi') d\xi' + \frac{y^2}{1-y} K_{2/3}(\xi) \right), \quad (4)$$

which in the classical regime ($\Upsilon \rightarrow 0$) reduces to the well known expression [1]

$$\frac{dW_\gamma}{d\omega\hbar} = \frac{\alpha}{\sqrt{3}\pi\gamma^2} \int_\xi^\infty K_{5/3}(\xi') d\xi'. \quad (5)$$

*This work was supported in part by the European Commission under the FP7 Capacities project EuCARD-2, grant agreement 312453, and by the Mexican CONACyT "BEAM" Programme

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The number of photons radiated per unit time is obtained by integrating over ω :

$$\frac{dN_\gamma}{dt} = \int_0^{E_e/\hbar} \frac{dW_\gamma}{d\omega} d\omega. \quad (6)$$

The number of photons emitted during a single collision can be obtained by integrating (6) in time and averaging over the bunch distribution, taking into account the variation of Υ . The result for head-on collision of Gaussian bunches is given in Ref. [6].

For all proposed high-energy circular colliders, Υ is much smaller than 1 (also see Table 1), and $\sigma_x \gg \sigma_y$. In this case we can approximate the average number of photons per collision as [6]

$$n_\gamma \approx \frac{12}{\pi^{3/2}} \frac{\alpha r_e N_b}{\sigma_x + \sigma_y} \approx \frac{12}{\pi^{3/2}} \frac{\alpha r_e N_b}{\sigma_x}, \quad (7)$$

and the average relative energy loss as

$$\delta_B \approx \frac{24}{3\sqrt{3}\pi^{3/2}} \frac{r_e^3 \gamma N_b^2}{\sigma_z (\sigma_x + \sigma_y)^2} \approx \frac{24}{3\sqrt{3}\pi^{3/2}} \frac{r_e^3 \gamma N_b^2}{\sigma_z \sigma_x^2}, \quad (8)$$

The average photon energy normalized to the beam energy, $\langle u \rangle$, is given by the ratio of δ_B and n_γ :

$$\langle u \rangle = \frac{\delta_B}{n_\gamma} \approx \frac{2\sqrt{3}}{9} \frac{r_e^2 N_b \gamma}{\alpha \sigma_z \sigma_x}. \quad (9)$$

In the classical regime, the average squared photon energy is related to the average photon energy via [1]

$$\langle u^2 \rangle \approx \frac{25 \times 11}{64} \langle u \rangle^2. \quad (10)$$

Noting that $\langle u \rangle \propto \int_0^{E_e} \omega (dW_\gamma/d\omega) d\omega$ and $\langle u^2 \rangle \propto \int_0^{E_e} \omega^2 (dW_\gamma/d\omega) d\omega$, we can use the general photon distributions (4) to check the applicability of this relation as a function of Υ . The validity of (10) up to $\Upsilon \sim 10^{-3}$ is illustrated in Fig. 1.

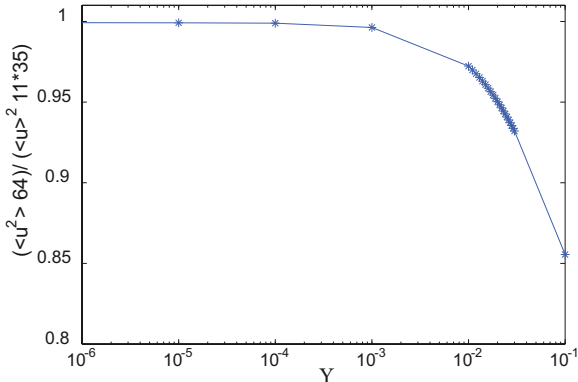


Figure 1: Mean square photon energy normalized by the square of the mean energy according to (10) versus Υ .

In the following we will need the excitation term $\{n_\gamma \langle u^2 \rangle\}$ for a single collision. According to (7) and (10), for small Υ this can be written as

$$n_\gamma \langle u^2 \rangle \approx 1.4 \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_z^2 (\sigma_x + \sigma_y)^3} \approx 192 \frac{r_e^5 N_b^3 \gamma^2}{\sigma_z^2 \sigma_x^3} \quad (11)$$

ENERGY LOSS AND DAMPING TIME

Using (8), and introducing the number of collision (interaction) points, n_{IP} , the average energy loss due to beamstrahlung per turn is given by

$$U_{0,BS} = n_{IP} \delta_B E_e \approx 0.84 \frac{n_{IP} r_e^3 E_e \gamma N_b^2}{\sigma_z (\sigma_x + \sigma_y)^2} \quad (12)$$

The longitudinal damping time in the presence of beamstrahlung is

$$\tau_{E,tot} = \frac{T_{rev} E_{beam}}{U_{0,SR} + n_{IP} U_{0,BS}} \approx \tau_{E,SR} \left(1 - n_{IP} \frac{U_{0,BS}}{U_{0,SR}} \right), \quad (13)$$

where T_{rev} denotes the revolution period, E_{beam} the beam energy, $U_{0,SR}$ the average energy loss per turn due to synchrotron radiation in the arc, and $U_{0,BS}$ the average energy loss due to beamstrahlung in one collision.

For all the proposed future circular colliders we have $U_{0,BS} \ll U_{0,SR}$, $\tau_{E,tot} \approx \tau_{E,SR}$, and also $\sigma_x \gg \sigma_y$. In the following we will assume these conditions to be fulfilled.

SELF-CONSISTENT ENERGY SPREAD

The energy spread increases due to the additional excitation from beamstrahlung at the collision point as

$$\sigma_{tot}^2 = \sigma_{\delta,SR}^2 + \sigma_{\delta,BS}^2, \quad (14)$$

where

$$\sigma_{\delta,BS}^2 = \frac{n_{IP} \tau_{E,SR}}{4T_{rev}} n_\gamma \langle u^2 \rangle \quad (15)$$

$$\approx 1.4 \frac{n_{IP} \tau_{E,SR}}{4T_{rev}} \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_z^2 \sigma_x^3} \quad (16)$$

scales as $1/(\sigma_x^3 \sigma_z^2)$. Introducing the parameter

$$A \equiv 1.4 \frac{n_{IP} \tau_{E,SR}}{4T_{rev}} \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_z^3} \quad (17)$$

and using the relation $\sigma_{z,tot} = \sigma_{\delta,tot} \sigma_{z,SR} / \sigma_{\delta,SR}$, self-consistency requires [10]

$$\sigma_{\delta,tot}^2 - \sigma_{\delta,SR}^2 = A \left(\frac{\sigma_{\delta,SR}}{\sigma_{\delta,tot}} \frac{1}{\sigma_{z,SR}} \right)^2, \quad (18)$$

where the subindex ‘‘SR’’ refers to the bunch length or energy spread computed with arc synchrotron radiation only. The explicit solution for the total energy spread is [10]

$$\sigma_{\delta,tot} = \left[\frac{1}{2} \sigma_{\delta,SR}^2 + \left(\frac{1}{4} \sigma_{\delta,SR}^4 + A \frac{\sigma_{\delta,SR}^2}{\sigma_{z,SR}^2} \right)^{1/2} \right]^{1/2}. \quad (19)$$

Solving (19) for the FCC-ee example parameters listed in Table 1, we obtain the shown values of $\sigma_{z,tot}$ and $\sigma_{\delta,tot}$.

Table 1: Example beam parameters for FCC-ee crab-waist (CW) collisions with a full crossing angle $\theta_c = 30$ mrad at the Z pole and at the WW threshold [11], as well as for possible operation on the Higgs resonance (62.5 GeV) in simple head-on (h.-o.) collision, and standard or pushed optimized monochromatization (m.-c.) [12], always considering $n_{\text{IP}} = 2$ identical IPs.

energy [GeV]	45.6	62.5	62.5	62.5	80
scheme	CW	h.-o.	m.-c.	m.-c.	CW
θ_c [mrad]	30	0	0	0	30
circ. C [km]	100	100	100	100	100
α_C [10^{-6}]	7	7	7	7	7
f_{rf} [MHz]	400	400	400	400	400
V_{rf} [GV]	0.2	0.4	0.4	0.4	0.8
$U_{0,\text{SR}}$ [GeV]	0.03	0.12	0.12	0.12	0.33
$U_{0,\text{BS}}$ [MeV]	0.5	0.05	0.01	0.01	0.21
τ_E/T_{rev}	1320	509	509	509	243
Q_s	0.025	0.030	0.030	0.030	0.037
N_b [10^{10}]	3.3	0.7	3.3	8.5	6.0
β_x^* [m]	1	1	1	0.25	1
β_y^* [mm]	2	2	2	1	2
D_x^* [mm]	0	0	0.22	0.11	0
$\epsilon_{x,\text{SR}}$ [nm]	0.09	0.17	0.17	0.17	0.26
$\epsilon_{x,\text{tot}}$ [nm]	0.09	0.17	0.21	4.16	0.26
$\epsilon_{y,\text{SR}}$ [pm]	1	1	1	1	1
$\sigma_{x,\text{SR}}$ [μm]	9.5	9.2	132	66	16
$\sigma_{x,\text{tot}}$ [μm]	9.5	9.2	144	323	16
σ_y [nm]	45	45	45	32	45
$\sigma_{z,\text{SR}}$ [mm]	1.6	1.8	1.8	1.8	2.0
$\sigma_{z,\text{tot}}$ [mm]	3.8	1.8	1.8	1.8	3.1
$\sigma_{\delta,\text{SR}}$ [%]	0.04	0.06	0.06	0.06	0.07
$\sigma_{\delta,\text{tot}}$ [%]	0.09	0.06	0.06	0.06	0.10
Υ_{max} [10^{-4}]	1.7	0.8	0.3	0.4	4.0
Υ_{ave} [10^{-4}]	0.7	0.3	0.1	0.2	1.7

SELF-CONSISTENT EMITTANCE

Non-zero dispersion at the interaction point (IP) may arise either due to optics errors or by design, e.g., for a monochromatization scheme [12]. In the presence of such IP dispersion, not only the energy spread increases due to the beamstrahlung, but also the transverse emittance. The dynamics is similar to the well-known effect of horizontal dispersion and conventional synchrotron radiation in the storage ring arcs.

We can formulate a set of equations for the longitudinal and transverse plane, which must be solved self-consistently for two unknowns: σ_x and σ_z .

The bunch length is related to the energy spread via [1]

$$\sigma_{z,\text{tot}} = \frac{\alpha_C C}{2\pi Q_s} \sigma_{\delta,\text{tot}}, \quad (20)$$

where C denotes the circumference, Q_s the synchrotron tune, and α_C the momentum compaction factor.

In the presence of a nonzero horizontal dispersion at the IP, the dispersion invariant [1],

$$\mathcal{H}_x^* \equiv \frac{(\beta_x^* D_x'^* + \alpha_x^* D_x^*)^2 + D_x^{*2}}{\beta_x^*}, \quad (21)$$

is nonzero, where β_x^* , α_x^* , D_x^* and $D_x'^*$ denote optical beta and alpha function (Twiss parameters), the dispersion and slope of the dispersion at the collision point, respectively. In this case the total emittance becomes

$$\epsilon_{x,\text{tot}} = \epsilon_{x,\text{SR}} + \frac{\tau_x n_{\text{IP}}}{4T_{\text{rev}}} \{n_\gamma \langle u^2 \rangle\} \mathcal{H}_x^* \quad (22)$$

where τ_x denotes the horizontal (amplitude) damping time due to synchrotron radiation, and the relative momentum spread

$$\sigma_{\delta,\text{tot}}^2 = \sigma_{\delta,\text{SR}}^2 + \frac{n_{\text{IP}} \tau_E, \text{SR}}{4T_{\text{rev}}} \{n_\gamma \langle u^2 \rangle\}. \quad (23)$$

These equations are coupled through the excitation term. Different simplified solutions can be obtained depending on whether $D_x^* \sigma_{\delta,\text{tot}} \ll \sqrt{\beta_x^* \epsilon_x}$, or $D_x^* \sigma_{\delta,\text{tot}} \gg \sqrt{\beta_x^* \epsilon_x}$.

For example, assuming $D_x^* \sigma_{\delta,\text{tot}} \gg \sqrt{\beta_x^* \epsilon_x}$, (e.g. case of monochromatization), $\tau_x = 2\tau_E$, and using (11) Eqs. (22) and (23) can be rewritten as

$$\epsilon_{x,\text{tot}} \approx \epsilon_{x,\text{SR}} + \frac{2B\mathcal{H}_x^*}{D_x^{*3} \sigma_{\delta,\text{tot}}^5} \quad (24)$$

$$\sigma_{\delta,\text{tot}}^2 = \sigma_{\delta,\text{SR}}^2 + \frac{B}{D_x^{*3} \sigma_{\delta,\text{tot}}^5} \quad (25)$$

with

$$B \equiv 48 \frac{n_{\text{IP}} \tau_E, \text{SR}}{T_{\text{rev}}} \frac{r_e^5 N_b^3 \gamma^2}{(\alpha_C C / (2\pi Q_s))^2}. \quad (26)$$

After solving Eq. (25) for the relative energy spread $\sigma_{\delta,\text{tot}}$, the emittance follows from Eq. (24). Using Eqs. (25) and (24) we obtain the total bunch length and emittance for the two cases of $D_x^* \neq 0$ (the second and third column) at 62.5 GeV in Table 1. A large horizontal emittance blow up due to beamstrahlung occurs for the ‘‘pushed optimized’’ monochromatization.

For the opposite case, $D_x^* \sigma_{\delta,\text{tot}} \ll \sqrt{\beta_x^* \epsilon_x}$, we find

$$\epsilon_{x,\text{tot}} \approx \epsilon_{x,\text{SR}} + \frac{2B\mathcal{H}_x^*}{\sigma_{\delta,\text{tot}}^2 \beta_x^{*3/2} \epsilon_{x,\text{tot}}^{3/2}} \quad (27)$$

$$\sigma_{\delta,\text{tot}}^2 = \sigma_{\delta,\text{SR}}^2 + \frac{B}{\sigma_{\delta,\text{tot}}^2 \beta_x^{*3/2} \epsilon_{x,\text{tot}}^{3/2}}. \quad (28)$$

The two equations (28) and (27) are coupled, and must be solved together. Equations (25) and (24) then yield the total bunch length and emittance shown in Table 1 for the three columns with $D_x^* = 0$.

The bunch length always follows from the relation (20).

ACKNOWLEDGMENTS

We thank A. Faus-Golfe, K. Oide, D. Shatilov, V. Telnov, and K. Yokoya for helpful discussions.

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